

Problem 2.9

[Difficulty: 2]

2.9 A flow is described by the velocity field $\vec{V} = (Ax + B)\hat{i} + (-Ay)\hat{j}$, where $A = 10$ ft/s/ft and $B = 20$ ft/s. Plot a few streamlines in the xy plane, including the one that passes through the point $(x, y) = (1, 2)$.

Given: Velocity field

Find: Plot streamlines

Solution:

Streamlines are given by $\frac{v}{u} = \frac{dy}{dx} = \frac{-A \cdot y}{A \cdot x + B}$

So, separating variables $\frac{dy}{-A \cdot y} = \frac{dx}{A \cdot x + B}$

Integrating $-\frac{1}{A} \ln(y) = \frac{1}{A} \cdot \ln\left(x + \frac{B}{A}\right)$

The solution is

$$y = \frac{C}{x + \frac{B}{A}}$$

For the streamline that passes through point $(x,y) = (1,2)$

$$C = y \cdot \left(x + \frac{B}{A}\right) = 2 \cdot \left(1 + \frac{20}{10}\right) = 6$$

$$y = \frac{6}{x + \frac{20}{10}}$$

$$y = \frac{6}{x + 2}$$

A = 10

B = 20

C =

	1	2	4	6
x	y	y	y	y
0.00	0.50	1.00	2.00	3.00
0.10	0.48	0.95	1.90	2.86
0.20	0.45	0.91	1.82	2.73
0.30	0.43	0.87	1.74	2.61
0.40	0.42	0.83	1.67	2.50
0.50	0.40	0.80	1.60	2.40
0.60	0.38	0.77	1.54	2.31
0.70	0.37	0.74	1.48	2.22
0.80	0.36	0.71	1.43	2.14
0.90	0.34	0.69	1.38	2.07
1.00	0.33	0.67	1.33	2.00
1.10	0.32	0.65	1.29	1.94
1.20	0.31	0.63	1.25	1.88
1.30	0.30	0.61	1.21	1.82
1.40	0.29	0.59	1.18	1.76
1.50	0.29	0.57	1.14	1.71
1.60	0.28	0.56	1.11	1.67
1.70	0.27	0.54	1.08	1.62
1.80	0.26	0.53	1.05	1.58
1.90	0.26	0.51	1.03	1.54
2.00	0.25	0.50	1.00	1.50

